International Journal of Community Currency Research

VOLUME 29 (2025) 1-27

NEUTRAL SETTLEMENT LAYER FOR INTEROPERABILITY BETWEEN DIFFERENT FORMS OF LOCAL MONETARY AND FINANCIAL EXPRESSION

Paolo Dini[1,2,3*] and Tomaž Fleischman[1]

[1] *Cycles Protocol SA,Avenue de l'Eglise-Anglaise 2, 1006 Lausanne, Switzerland [paolo,tomaz]@cycles.money*

[2] *ecoloc, St. Alban-Vorstadt 19, 4052 Basel, Switzerland*

[3] *School of Physics, Engineering and Computer Science, University of Hertfordshire, Hatfield, AL10 9AB, UK*

[*] *Corresponding author*

ABSTRACT:

The paper addresses the late payment problem in B2B networks by multilateral set-off of mutual debts. It introduces as innovations in accounting 1) future commitments in addition to existing obligations and 2) a new notation to represent balance sheet relationships as network edges. It builds on them to develop 3) the concept of multilateral settlement flow as an innovation in the payment system. These innovations, in turn, lead to significant financial innovations: 4) payment system neutrality, 5) monetary and financial expression, and 6) interoperability between non-convertible currencies. The paper asks two linked research questions: whether these innovations can increase a) the macroeconomic impact of local currencies, and b) the ability of economic systems to meet sustainability expectations. A positive answer to both questions is argued through a "toy model" of a network. We also simulated the presence of clusters of companies transacting with inside money based on a dataset of more than 40,000 companies. Up to 95% of the total Net Internal Debt can be settled within the communities, with the remainder settled with outside money. The implication is that communities with sufficiently high internal trust will be able to use their own inside money and, therefore, to interact economically and financially with other communities and the fiat economy from a position of greater market power. Similarly, the expression of financial relations such as p2p loans implies the ability to take charge of their financial needs with lower reliance on extractive third parties operating in outside money.

KEYWORDS:

Multilateral Settlement, Collaborative Finance, SMEs, Trade Credit, Neutral Settlement Layer, Blockchain

To cite this article: Dini and Fleischman (2025). 'Neutral Settlement Layer for Interoperability between Different Forms of Local Monetary and Financial Expression' INTERNATIONAL JOURNAL OF COMMUNITY CURRENCY RESEARCH - VOLUME 29 (2025) 1-27; https://www.ijccr.net/; ISSN 3106-0781; DOI https://doi.org/10.26034/zh.ijccr.2025.8310

1. INTRODUCTION

In light of the limited macroeconomic effect of local currencies, the functionalist research question asked by this paper is whether multilateral settlement as a payment system innovation can increase the positive impact of local currencies on local economies. However, since the motivation for setting up most local currencies in the first place is economic sustainability of the weakest members of the economy, several of the innovations we present are also aligned with the sustainability agenda, where by the latter we could invoke the UN's Sustainable Development Goals (SDGs) or a more generic understanding of economic, social, environmental, and democratic sustainability. A second, deeper research question, therefore, is whether the innovations we are presenting can align the functionalist economic goal above with normative sustainability goals in one or more of its forms. More precisely, throughout the paper we argue that some of the structural innovations we are proposing strengthen not only the economic impact of local currencies but also the ability of economic systems to meet sustainability expectations.

A challenge in demonstrating this claim and communicating this vision, however, is that our analysis of how and why the innovations we propose will be effective in this manner is based on a deep integration of very different disciplines. In particular, the most effective way to describe the payment system innovation we propose requires innovations both in the language of accounting *and* in what is being accounted. These innovations, together, enable financial innovations which add to the economic and sustainability impact of local currencies. As accounting practice is closely linked to legal concerns, they too play an indirect role in this paper. Since direct empirical evidence of the effects of some of these innovations is lacking, the paper provides simulations of a large network of real companies with real invoices as a basis for arguments in support of the expected effects.

In more detail, switching from sequential, bilateral transactions to simultaneous multilateral settlement requires a significant leap in imagination and relies on graph theory concepts that are closer to mathematics than to economics or finance. The effective application of multilateral settlement over networks of transacting parties requires significant innovation in the language and analysis tools of accounting, especially since the most relevant type of transactors to economic development are firms, for which accounting practice and processes are essential. At the most fundamental level of the monetary instruments themselves, we believe that mutual credit offers the greatest potential for a transition to sustainability because distributing the power to create money to the participants represents a radical *political* innovation even before being a monetary, financial, or economic innovation. Although we offer some considerations based on a social science perspective in support of the relevance of this view to the sustainability agenda, the main argument relies on the expected effects of 'inside money' – i.e. money issued locally by the participants themselves, or "trust-based" money – on the financial self-determination and market power of networks of firms. This claim is supported by payment system simulations of a large network of real firms with real invoices that quantifies the relative volume of payments effected with inside vs. outside (i.e. fiat) money using different network statistics to define different types and number of sub-communities.

Thus, the paper is organized as follows. After a brief methodological note in Section 2, the motivational context-setting discussion of Section 3 draws mainly on sociology, economic anthropology, monetary theory, and political theory to explain the importance of local currencies to sustainability. The description of the payment system innovation here dubbed 'multilateral settlement', which is described mainly in our prior work (Fleischman et al., 2020; Fleischman and Dini, 2021) and is only briefly summarized in Section 3, relies equally on graph theory, which is a branch of mathematics, and on accounting. After a brief historical summary of the double-entry book-keeping method, the innovations in accounting introduced in Section 4 consist in a new graphical language to visualize different kinds of stocks and flows that relies on graph theory and on the explicit introduction of a time dimension that puts future commitments and past obligations on the same mathematical footing. The quantification of the economic effect of these innovations through the simulation of the multilateral settlement flow over a large network of real firms presented in Section 5 requires a mixture of statistics and software engineering. Finally, the discussion of the effects on the economic impact of these innovations that is presented in Section 6 is informed by finance, social science, and political theory.

2. METHODOLOGICAL NOTE

In our own experience, it is not possible to understand the enormous complexity of the social, economic, and political processes constituting any economic system through a single disciplinary viewpoint or language. This calls for a "methodology of convergence" between different disciplinary languages and, more fundamentally, epistemologies. For example, the qualitative discussion about the social and political relevance of mutual credit could benefit from a more positivist engagement with empirical observation and measurements of economic data; and the truth value of our mathematical conclusions about the efficiency of the settlement system we are presenting does not strictly require but would benefit from empirical validation.

Although we cannot claim to have arrived at even an outline of such a methodology, the increasing need worldwide for a transition to sustainability is providing an organizing principle that helps make sense of the very complex systems and sub-systems involved. And the focus on local currencies is starting to make such a daunting vision actionable. Therefore, while significant switches in disciplinary and even epistemological registers are necessary when moving between different sections of the paper, the joint applied endeavour to achieve environmental, social, economic, and democratic sustainability at the local level can help reach a useful integration and convergence – if not the unification (Dini et al., 2011) – of different disciplinary viewpoints. More specifically, this paper presents a wire-frame for practical intervention that acknowledges the complexity of the challenge of sustainability and is opportunistic in terms of efficiency and effectiveness. In particular, we focus on the graph structure of the payment system and on the corresponding accounting innovations needed to implement it efficiently in present-day business contexts while enabling the interoperability of local currencies with the wider economy.

3. MOTIVATIONAL CONTEXT AND LITERATURE REVIEW

Over the past 50 years inequality has been steadily increasing (Piketty, 2011). This phenomenon is correlated with a drain of liquidity from the real towards the financial economy (Arrighi, 2001; Stevenson, 2024; Simmons et al., 2021). More generally, our cultural, social, and environmental spaces are being depleted as everything is commodified and turned into money (Eisenstein, 2011). A consequence of the dearth of real economy liquidity is the problem of late payments (Walker, 2017), which disproportionately affect small and medium-sized enterprises (SMEs). Surprisingly, the banking sector itself is also affected. In fact, as the consumer savings and the company working capital in peripheral economies dwindle, local and community banks fail to meet the capital requirements mandated by the Basel accords (ECB, 2023b), and are forced either to consolidate into bigger banks or be absorbed by existing multinational banks. Therefore, community banks are disappearing all over the world, with significant consequences for access to credit by SMEs (Padgett, 2013; ECB, 2023a).[1]

The problem of liquidity drain is more acute for smaller, local economies. As a consequence, in many different countries and historical periods local, community, and complementary currencies have emerged as a bottom-up response to financial crises and the flight of liquidity from weak economies at the margins towards financial centres (Greco, 2009; Gómez and Dini, 2016). Among the several different kinds of local currencies (Blanc, 2011), in our opinion zero-interest mutual credit is the most effective at addressing liquidity drain, especially when it is non-convertible. The combination of local use and zero interest increases its velocity of circulation,[2] which is a symptom of a more dynamic local economy.

In addition, mutual credit is the most democratically radical form of complementary currency because it distributes the power to create "money" to all the circuit members by allowing them to issue their own debt as 'credits' that are transferable within the circuit.[3] The backing for the credits issued in this manner consists of the future products and services sold by members within a set time period following each issuance (e.g. a year) (Greco, 2009). This has implications that are both economically practical and transformative on the level of subjective perceptions. For example, when the members are SMEs, as in the WIR[4] (Studer, 1998) and Sardex (Sartori and Dini, 2016; Littera et al., 2017; Dini and Kioupkiolis, 2019) systems,[5] rather than individuals, as in the LETS case (Croall, 1997), the result is a more economically sustainable circuit. This is visible through the ability of the circuit manager to get paid for its services rather than relying on voluntarism to different extents as in LETS, which often leads to burn-out (Croall, 1997). But in both cases the feeling of empowerment experienced by the users is unmistakable: in the LETS case lowering the threshold of employability can have a very positive effect on feelings of self-worth, while for SME-based circuits access to zero-interest working capital in place of dependence from the banks is very liberating (Sartori and Dini, 2016).

In spite of its very positive characteristics, mutual credit tends to be limited in terms of scalability and macroeconomic impact, the only exception arguably being the beneficial counter-cyclical effect of WIR on the Swiss economy (Stodder, 2009); but even then the largest impact was achieved in 1993 when the WIR transaction volume reached 0.8% of Swiss GDP.[6] Similarly, in 2018 the Sardex transaction volume reached 0.5% of private-sector Sardinian GDP (Dini and Kioupkiolis, 2019). The limited impact is partly due to the fact that non-convertibility makes import-export much more difficult if not impossible (where these terms refer both to geography and to currency areas), which in turn slows the growth of the circuit. The ability to perform mixed trades in credit and fiat (Littera et al., 2017) does increase the impact, but not to a macroeconomically relevant level (Studer, 1998; Dini and Kioupkiolis, 2019).

In this paper we address this problem by building on our previous work to define a 'neutral settlement layer' protocol – and a corresponding new network graphics notation – that generalizes the effect of multilateral trade credit set-off (MTCS) (Simić and Milanović, 1992; Eisenberg and Noe, 2001; Fleischman et al., 2020) through a more general application of the *same* graph theory algorithm. This more general network flow protocol enables interoperability across non-convertibility boundaries, thereby opening local circuits to the benefits of interacting with each other and with the wider economy without, however, exposing them to the drawbacks of extractive finance. As multilateral, *simultaneous* (Block et al., 2025) clearing of trade credit obligations accumulated within a network of companies over a set period (e.g. a month) as well as of future contractual relationships called 'commitments', the neutral settlement protocol can be seen as complementary to bilateral, *instantaneous* clearing based on transactions. It is an alternative, and more efficient, way to make use of the available liquidity.

Figure 1: *Components of simultaneous multilateral settlement compared to instantaneous transactions*

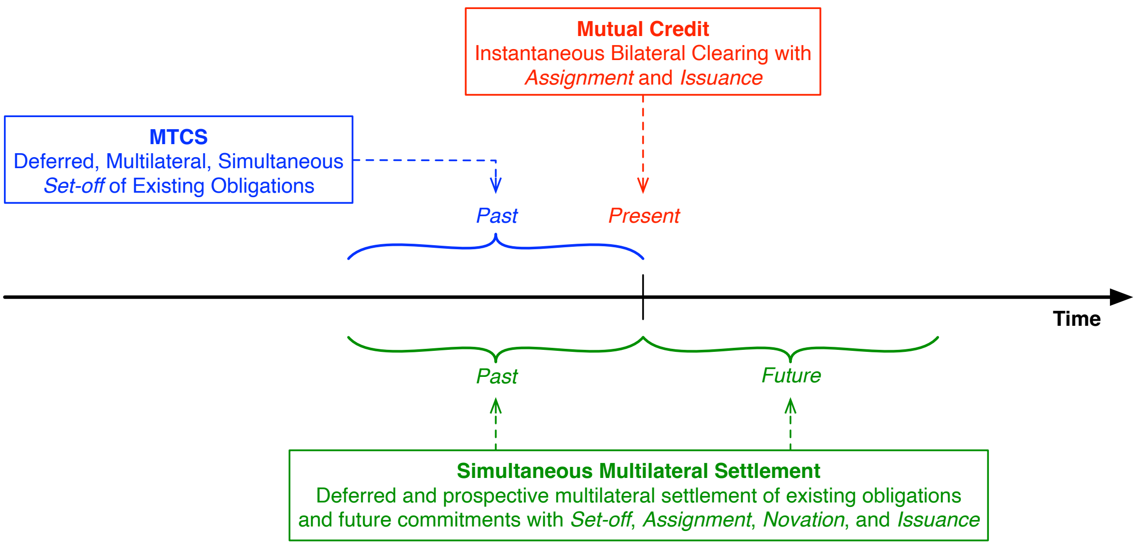


Figure 1 shows how the introduction of a time axis clarifies the relative roles of MTCS and mutual credit with respect to settlement and in particular relative to the generalized settlement layer we are presenting here. The curly brackets indicate the time windows during which the obligations and/or commitments are accumulated to create the interaction network (to be defined below). While all three settlement methods shown are applied at one instant, shown as 'Present' in the figure, mutual credit transactions stand separately from simultaneous multilateral settlement. In spite of this difference in protocols, from a social science point of view as suggested in Section 5.2 of Giaglis et al. (2021) the principles and practice of mutual credit and MTCS fall under the same theoretical framework as different instances of 'collaborative finance' (CoFi).

From a social science point of view, both sets of services build on the concept of 'inside money' (Kiyotaki and Moore, 2000; Lagos, 2010; Brunnermeier et al., 2021). In the standard economics literature, inside money is credit money issued by banks while 'outside money' is central bank reserves. By analogy, we regard endogenously-generated local currencies like mutual credit a form of inside money, where the role of outside money in this case is played by bank money (fiat). In general, inside money arises when the trust needed to issue debt as a currency is extended within a given community, so that each member of that community can issue its own debt that will be accepted as transferable IOUs[7] by the other members. If the community in question cannot conjure enough trust, then it is obliged to use outside money, i.e. money that was issued by agents outside the community. Outside money is always an asset of community members, never a liability – a fact that contributes to the everyday perception of bank money as an asset or precious commodity.

For the sake of completeness, we mention that although mutual credit and MTCS address the dearth of *working capital* in networks of SMEs and the difficulties they encounter accessing credit from banks, the CoFi principle of generation of economic value through collaboration is also compatible with innovative forms of the commons economy that address *investment capital*, such as the Housing Commons (Darby et al., 2023) and Energy Commons models (Darby, 2020, 2022). These other forms of collaborative finance will not be discussed in this paper due to space limitations.

3.1 Brief Description of the Multilateral Set-off Concept

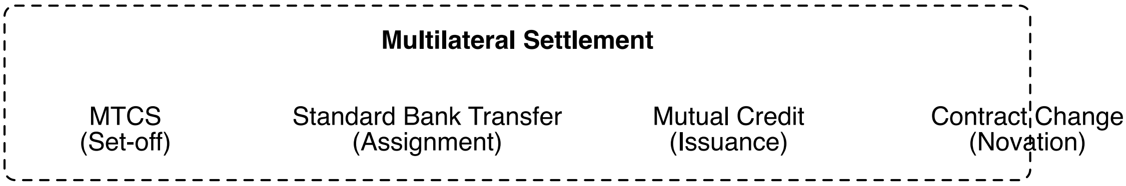
For readers unfamiliar with the Multilateral Trade Credit Set-off (MTCS) algorithm described in Fleischman et al. (2020) and Fleischman and Dini (2021), this section provides a very succinct summary of the idea. Bilateral set-off is a well-known accounting operation (UNIDROIT, 2016) between mutually indebted economic entities (usually firms) where the smaller obligation (e.g. an invoice) is subtracted from the larger, leaving a smaller obligation in only one direction and, therefore, requiring less liquidity overall to settle both contracts. MTCS generalizes this idea to three or more firms forming a closed directed cycle of obligations by subtracting the smallest from all the obligations in the cycle. Given a set of transacting firms, the set of obligations between them forms a directed graph or network, where the nodes are the firms and the edges or arcs are the obligations. The MTCS algorithm finds all the closed cycles in such a network, which could involve hundreds of thousands of firms and millions of obligations, and subtracts the smallest obligation amount from each cycle in such a way that the maximum possible liquidity-saving is guaranteed at the overall network level. Of course, in order to build up such a network some time is needed, e.g. one month for B2B contexts. Thus, MTCS is a particular type of deferred (partial) settlement system which, at the time it is applied, only takes into consideration obligations accrued in the network over a time-period just completed (usually 1 month).

By introducing liquidity sources (such as a bank) as nodes in the network, additional types of obligations can be modelled in the same way, such that in its most general form the same algorithm can settle also assignment, issuance, novation, and loans-related obligations and commitments in the same 'atomic' event.[8] This is the essence of the multilateral settlement flow presented in this paper.

3.2 Initial Definitions

Since the terminology used in this paper may be unfamiliar and since there are overlaps between at least two terms, Figure 2 shows a diagram of what we mean by 'multilateral settlement' relative to other commonly used terms and, in parentheses, their corresponding accounting terms. This diagram is only meant to orient the reader, these terms are explained further at different points in the paper.

Figure 2: *Relationship between key concepts and terms used in the paper*



Novation is only partly included in our multilateral settlement model because when it is effected as part of a clearing house's operations it cannot be included in the simultaneous settlement operation. Another kind of multilateral settlement operation, called 'netting', is a form of simultaneous clearing that guarantees the maximum possible liquidity-saving but at the possible expense of changing the direction of one or more obligations; each change corresponds to a change of contract – hence novation – that introduces significant admin overhead and that is therefore not desirable from a user experience and adoption point of view (MTCS guarantees the maximum possible liquidity-saving without any contract changes). The kind of novation MTCS supports extends bilateral novation to a network context.

The paper is concerned with settlement of any obligation, whether it be trade credit in the real economy or a loan in the financial economy. As shown in Table 1, we define four axes of analysis along which it is convenient to identify four pairs of opposing binaries, which afford a simpler and more accessible presentation.

Table 1: *Conceptual binaries along the four main axes of analysis of settlement systems*

Axis of Analysis	**Prevailing Theory and Practice**	**Extension or Generalization**
Agent (node)	Payer's perspective	Payer and payee on equal footing
Network Topology	Bilateral *transaction* (could be intermediated, e.g. through a clearing house)	Multilateral settlement *flow*
Interaction (edge)	Past (→ obligations)	Past and future on equal footing (→ obligations & commitments)
System State	Stock-flow analysis	Network flow state before and after settlement flow is applied

Financial Networks can be represented as connections between balance sheets. Balance sheets and the method of double-entry book-keeping allow four ways to settle an obligation (Clavero, 2022).[9] Accounting practice is focused on achieving settlement through the *bilateral* transfer of titles to assets, i.e. assignment. By formalizing the settlement of existing obligations, current accounting practice can be said to reflect the perspective of the payer and to be focused on the past. But there are three other operations that are just as important for reaching settlement – set-off, novation, and issuance (Clavero, 2022) – which can be effected also through *multilateral* processes and algorithms that include also the payee's perspective and are focused on the future:

- **Set-off** is the core of the basic MTCS algorithm.
- **Assignment** involves the transfer of title to an asset and is by far the most common type of transaction.
- **Novation** involves a change in the contract between the transacting counterparties.
- **Issuance** can take many forms, but the easiest to understand in the real economy is mutual credit. In the financial economy the most common example is the disbursement of loan proceeds.

This paper introduces a generalization through which the connections between balance sheets are called 'interactions', where an interaction is called an 'obligation' if it is already part of an existing contract and is therefore concerned with the past, or a 'commitment' if it refers to an intent and is therefore concerned with a future contractual relationship. Networks involving obligations and/or commitments are called 'interaction networks'. It is important to note that these network generalizations transcend existing legislation, which is limited to bilateral transactions, and will therefore require a cover contract with the service provider.

Ultimately, finance is about building a bridge towards the future. Adding future-commitment edges enables the emergence of liquidity that would otherwise remain "hidden" in the topology of the network. Rather than 'issuance', (which is a bilateral transaction), we refer to this effect as the 'expression' of new financial relations that, by participating in the settlement flow, enable a more efficient use of the available liquidity and the clearing of more debt in the network as a whole. Since settlement happens at a single point in time, there is no notion of time in a multilateral settlement flow. Therefore, its application can resolve both obligations and commitments at the same time, as an atomic event.

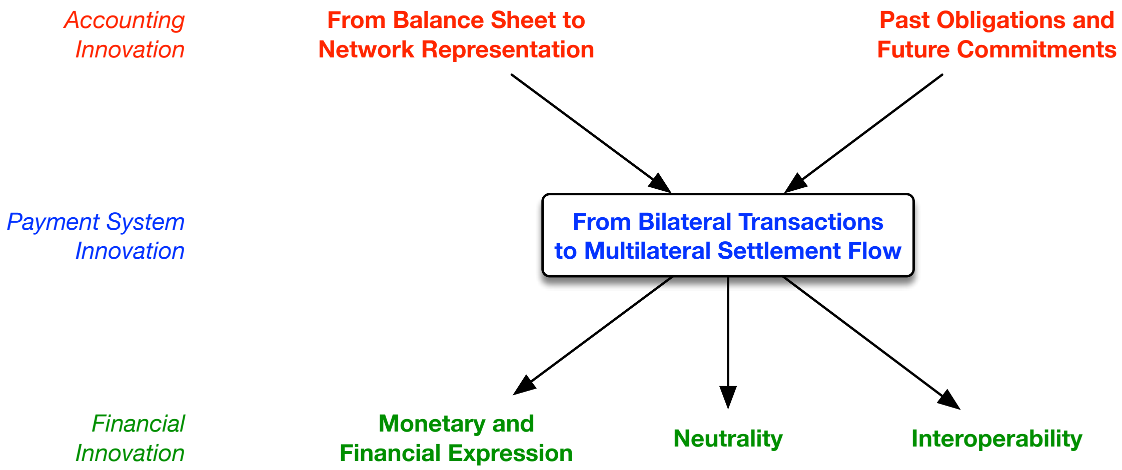
3.3 Conceptual Framework

While the terminology for the four types of settlement above is well-established, for the four additional generalizations listed on the right of Table 1 practice is leading codification and formalization. In particular, current accounting practice lacks the representation of flows from future intended or agreed contracts. Therefore, in this paper we propose a general framework for settlement flows along with a generalization in the language of accounting in the form of the new term 'commitment', which is related to a future obligation, and as a graphical notation that includes the intended promised flows in the financial network.

Multilateral settlement is a strong structural determinant of neutrality in payment systems because it strengthens the interoperability between different forms of inside money (e.g. mutual credit) and outside money (e.g. fiat, crypto) and, therefore, the market power of local economies. This can be seen from four points of view, as follows.

From the technical design perspective the neutrality of multilateral clearing stems from the fact that it does not settle sequentially by bilateral transaction but atomically over the whole network at once. Breaking down the settlement of a complex network into sequential transactions favours those who have the tools and the power to prioritize or delay their transactions. By contrast, multilateral clearing methods emphasize the objective function of the solver, and MTCS maximizes the global settled amount across the whole network. Depending on who controls the objective function, alignment with community values is not guaranteed. However, the participation of different stakeholders implied by the CoFi approach provides some level of protection through transparency and governance. From the political economy and monetary theory perspective, multilateral clearing is neutral because it is agnostic about the market power of the participants, and because, by operating only on the unit of account, it is also agnostic about cultural or political choices affecting the store of value or the medium of exchange functions of money. From the accounting perspective, the innovation consists in representing the four operations above using only two accounting primitives: *obligations*, focused on the past, and *commitments*, focused on the future. From the accounting perspective, therefore, this generalization of accounting concepts and language supports neutral settlement because by formalizing multilateral settlement in the simplest possible way it makes all four ways to settle equally accessible regardless of the sophistication of the finance departments of the participating companies, thereby achieving a democratizing effect. Finally, from the perspective of the interoperability between regions and currency areas, our multilateral settlement protocol is neutral because, as already mentioned above, it enables weaker/local economies to access the benefits of trading with the wider economy while minimizing the predatory effects of extractive finance (e.g. factoring services offered by large banks). The interdependence of these ideas is shown in Figure 3, which also reflects the logical (rather than narrative) structure of the paper.

Figure 3: *Conceptual map and logical structure of the paper*



The paper, therefore, has several theoretical and applied interlocking aims. Building on our previous work (Fleischman et al., 2020; Fleischman and Dini, 2021), Section 4 introduces the accounting language of obligations and commitments to unify the four ways to settle not only for bilateral transactions but also for multilateral settlement flows. Section 4.4 steps through a detailed "toy example" that shows how the new terms and concepts are used in practice from a purely technical/accounting point of view. Section 5 uses simulations on a real data set of more than 40,000 Italian companies to demonstrate the impact of multilateral settlement on the relative volumes of inside and outside money controlled and managed by communities, with implications on their market power and financial expression. Section 6 retraces the main points of the argument and offers some conclusions.

4. OBLIGATIONS AND COMMITMENTS

4.1 Historical Background

The balance sheet equation has been written since the Middle Ages as

$$\text{Assets} = \text{Liabilities} + \text{Equity}. \quad (1)$$

It is easier to understand if 'net worth' is used as the alternative name for 'equity':

$$\text{Net Worth} = \text{Assets} - \text{Liabilities}. \quad (2)$$

The reason it is not written in this more intuitively accessible way is that negative numbers were unknown in the Middle Ages. So Medieval accountants in general and whoever invented double-entry book-keeping in particular had to come up with a way to make things work even without negative numbers.[10] In upcoming figures and tables we continue to use 'equity' because it is the more common term.

Regardless of Medieval mathematicians' awareness or opinion of negative numbers, it seems unlikely that more prosaic professions like traders and their accountants were aware of them or used them in the way we use them today. Therefore, the discovery that double-entry book-keeping makes use of an elementry construction in modern algebra to represent the signed integers is all the more remarkable. Ellerman (1985) recounts that Arthur Cayley was the first mathematician to notice in 1894 the link between double-entry book-keeping and the algebraic group of differences, which had itself first been published by Sir W R Hamilton in 1837.

It is not known where or how double-entry book-keeping originated. Although the accountants of Venice, Milan, Genova, and Florence are usually credited with its development (Antinori, 2004), it could have originated in the Far or Middle East and arrived in Europe at a time in the late Middle Ages when the Italian Merchant Republics (Amalfi, Pisa, Genoa, and Venice) and other city states were dominant (900-1500 AD, approximately). The use of a mixture of Latin and early Italian in the recording of transactions of many small merchants and shop-keepers from different parts of Italy, however, suggests that the basic concepts might indeed have emerged in the Italian city-states, aligning later with practices from farther afield. Indeed, according to Sangster (2025), 'The evidence we have ... indicates that the origins of double entry lie in the 12^{th} century regional fairs of northern Italy'. Another reason for the attribution is that the first detailed description of the method was published by the Italian mathematician and Franciscan friar Luca Pacioli in 1494 (Geijsbeek, 1914). Be that as it may, the method notoriously seems convoluted, counter-intuitive, and opaque to the non-initiated. Although we can now see why the balance was written with Liabilities on the other side of the equation, it is remarkable that this practice has remained stable in accounting in spite of the huge advancements in mathematical thought over the past five centuries.

There is another layer of opaqueness to double-entry book-keeping, caused by the fact that adding to the Assets column is called a 'debit' operation, while adding to the Liabilities is called a 'credit'. This is confusing because the modern default assumption is that Assets and Liabilities refer to the owner of the balance sheet, such that we would expect adding to the assets to be a credit, etc. But this is not how the recording of credits and debts originated in Medieval Italy. In modern-day Italian accounting terminology, the term for 'debit' is 'dare', to give, and for 'credit' is 'avere', to have. These terms are as counter-intuitive in Italian as they are in English. The solution to this puzzle can be found in Sosnowski (2003), who provides two examples of 'dare' and 'avere' journal entries from 1211, written in a mixture of Latin and Florentine dialect. The examples Sosnowski cites are illuminating in showing how the language was used to describe and record these positions. In the following, we use 'Alice', 'Bob', and 'Charlie' for simplicity.

To wit, then, the first example says that Alice 'deve dare', i.e. 'must give', 23.5 Lire to Bob. In a modern balance sheet, this would be listed in Bob's 'Dare' column, which is translated to 'Debit' in English. Since Bob is supposed to *receive* this money, it makes sense for this to be an Asset in modern terminology. The second example says that Charlie 'deve avere', i.e. 'must have' or 'must receive', 4 Lire from Bob. This would be listed in Bob's 'Avere' column, which is translated as 'Credit' in English. Since Bob is supposed to pay this money to Charlie, it makes sense for this amount to be his Liability in modern terminology.

We can therefore conclude that the actual meanings of the terms debit and credit as used in book-keeping are *exactly the opposite* of their modern intuitive interpretation based on how a balance sheet works. And the reason for this is that the explicit reference to the other transactors was dropped early on along with the original 'must' in 'must give' and 'must receive'. Without the beginning of the sentence, the terms indeed appear to refer to Bob's actions rather than to the other transactors'. However, the specialized accounting meaning remained dominant and replaced the common meanings of these terms, causing endless confusion in countless minds ever after. In other words, the Dare and Avere terms, or the Debit and Credit terms in English, make sense as long as we realize that they refer to the other people or companies with which the company whose balance sheet we are looking at is transacting.

Ellerman (1985) explains that 'double-entry' book-keeping does not refer to the writing of the same amount on both sides of the balance sheet equation because the fact that two accounts on opposite sides of the equal sign are affected is just the application of simple algebra, which ensures that the original equation remains an equation. Rather, he argues that the term refers to a system of rules for *recording* transactions as a method that makes it easier to avoid nonsensical entries such as adding to the Liabilities account and the Equity account at the same time. Debit entries add to debit balance accounts and subtract from credit balance accounts, and conversely for Credit entries. It seems more correct to say, however, that combining these operations as a double entry according to specific formal rules provides a "built-in" way to ensure that proper book-keeping operations are being performed consistently with the algebraic properties of equations, but without having to understand the latter.

With this background, hopefully the discussion that follows on how the settlement of obligations is recorded in balance sheets will be easier to understand.

4.2 Four Ways to Settle

Clavero (2022) explains that there are four ways to settle an obligation. We modify his presentation slightly without changing the essence. Table 2 shows an example scenario in which a Payer (debtor, buyer) settles an obligation of € 100 towards a Payee (creditor, seller). This is shown by the fact that the Accounts Payable (AP) of the Payer and the Accounts Receivable (AR) if the Payee are both decreased by 100. The circled letters in the first row refer to four possible ways in which this transaction can be settled, which is explained in detail in the following.

Table 2: *Two balance sheets showing a settlement transaction*

	Payer				**Payee**		
	Assets	Liabilities	Equity		Assets	Liabilities	Equity
ΔFunds	Ⓐ	Ⓑ		ΔFunds	Ⓓ	Ⓒ	
ΔAP		€ −100	€ +100	ΔAR	€ −100		€ −100

Before we get into the quantitative details of the four ways to settle, it helps to provide an intuitive explanation of each, as shown in Figure 4. The four diagrams can be described as follows:

- **Set-off** assumes a pre-existing obligation from the Payee towards the Payer. If the Payer generates a new obligation towards the Payee, the first obligation can be 'redeemed', or used to set-off the newer one. This is a bilateral set-off example. MTCS applies the same concept from two to any number of counterparties. The visualization shown in Figure 4 involves two equal invoices, whose set-off results in no obligations remaining. More generally, the smaller amount is subtracted from both, leaving the difference as a smaller obligation in only one direction.
- **Assignment** is just a standard payment from a debtor to a creditor. However, since from the banking system's point of view a deposit is a liability towards the client holding that asset, assignment in effect results in the substitution of one creditor with another.
- **Novation** is the opposite: it involves the movement of a person's funds between different banks. More generally, novation refers to any change in contract.
- **Issuance** can take different forms. In mutual credit, it replaces an existing obligation with the issuance of the Payer's liability (an IOU) in the form of a debt towards the circuit. The Payee's positive balance is a credit towards the circuit.

Each quadrant of Figure 4 details the 'before' and 'after' states of the settlement transaction for the four ways in which it can be completed. In each case, the transaction causes a change in the relationships between the counterparties whose description is shown in parentheses. In this case, the symbol Δ shown refers to this topological change rather than to the change in the net position[11] of each participant or to the network flow state, to be defined in Section 4.3.1. While for set-off and issuance the change affects creditor and debtor equally, for assignment and novation it does not. In the latter two cases, the description refers to the point of view of the shaded nodes.

Figure 4: *Visualization of the four ways to settle an obligation according to (Clavero, 2022)*

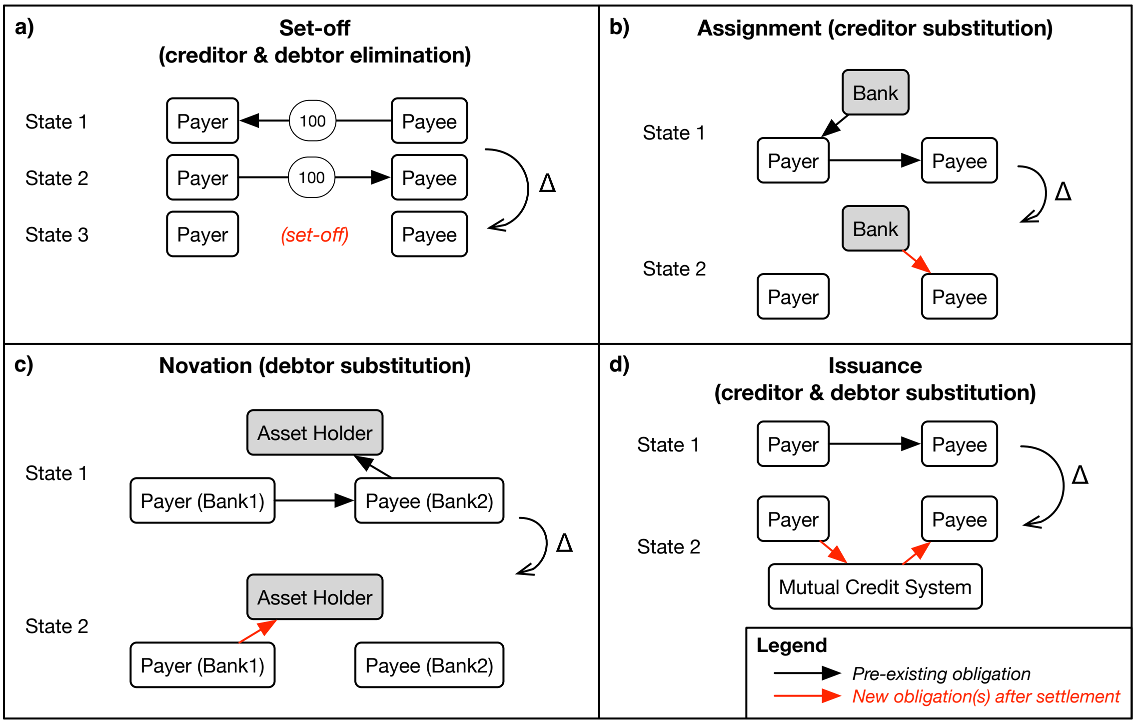


Table 3 shows a more condensed representation of the same four operations as changes to be applied to the balance sheets of the Payer and the Payee.[12] A value of 100 is used for all amounts for simplicity. For each of the four different ways in which an obligation can be settled, the table shows that the change in accounts payable and accounts receivables is the same, as we would expect since the table refers to the same obligation. Section 4.3.2 discusses the effect of settlement on the participants' balance sheets in more detail.

Table 3: *Four ways to settle an obligation, detailed balance-sheet view*

		Payer			Payee		
		Ⓐ	Ⓑ		Ⓓ	Ⓒ	
		Assets	Liabilities	Equity	Assets	Liabilities	Equity
Set-off (A-C)	ΔFunds	−100		−100		−100	+100
	ΔAP/AR		−100	+100	−100		−100
Assignment (A-D)	ΔFunds	−100		−100	+100		+100
	ΔAP/AR		−100	+100	−100		−100
Novation (B-C)	ΔFunds		+100	−100		−100	+100
	ΔAP/AR		−100	+100	−100		−100
Issuance (B-D)	ΔFunds		+100	−100	+100		+100
	ΔAP/AR		−100	+100	−100		−100

Finally, Table 4 shows a more compact way to represent the four ways to settle as a 2×2 matrix that makes use of the circled letters in Table 3 to indicate which accounts are affected. In this table we have swapped the two columns, relative to Clavero's rendition, to diagonalize the matrix algebraically without changing the essence of what it says.

Using balance sheets and the logic of four ways to settle one can easily depict more complex settlement scenarios with one or more intermediaries. When the number of interconnected balance sheets is high enough to form a network (Bardoscia et al., 2021), however, it is often difficult to follow the settlement of multiple obligations. So it makes sense to develop a more compact network graphical representation. The result will be two-fold:

- generalization of a bilateral settlement transaction to a 'settlement flow' which applies to a multilateral obligation network
- generalization of the settlement of existing obligations focused on the past to a settlement flow that includes also commitments focused on the future.

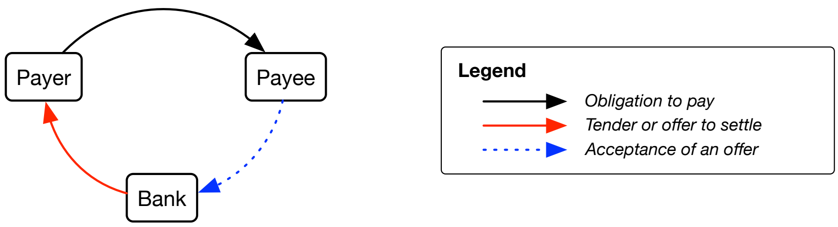
Table 4: *Compact summary of four ways to settle, after Clavero (2022)*

		Payee	
Payer	Payment by:	Ⓒ = € −100	Ⓓ = € +100
	Ⓐ = € −100	Set-off	Assignment
	Ⓑ = € +100	Novation	Issuance

4.3 *From Balance Sheet to Network Representation*

The first step in developing a network-centric graphical notation is to make the presence of the liquidity source (e.g. a bank) an explicit and integral part of the network. Figure 5 shows an example of how this can be set up. The fact that a Payer has some amount of money at its disposal, i.e. *already existing* in its bank account, is shown as a solid red arrow from the Bank to the Payer, signifying an obligation because a current account is a liability on the Bank's balance sheet. The black arrow is a *pre-existing* obligation from Payer to Payee. And the dotted blue arrow represents the Payee's willingness to accept the funds into its bank account *at some future time*. In this depiction, the 'Bank' node stands for a single bank or, more generally, the banking system as a whole.

Figure 5: *Example of graphical notation for three interaction types*



4.3.1 *The Network View*

This notation is now used to represent the four ways to settle as interaction networks.[13] First, as a notational simplification, 'Payer' and 'Payee' are treated as network nodes, denoted by v_1 and v_2 (where the v is a standard graph theory variable name that stands for 'vertex'). Second, another simplification is to call the Bank simply S, which could stand for 'source' in 'liquidity source' but is actually laying the ground for two layers of abstraction: first, it could represent different types of liquidity sources, e.g. a mutual credit system, a bank, a generic overdraft facility, etc; second, multiple instances of the liquidity source node can serve as different settlement media (currencies) being used *simultaneously* in the same interaction network – something that MTCS can take in stride.

Figure 6 shows the result, which requires some explanation because the solid blue arrow and the dotted red arrow have not been discussed yet. The solid blue arrow represents an *existing* obligation towards the liquidity source, such as a loan to be repaid. The dotted red arrow, on the other hand, stands for a credit line and represents a commitment to make (a certain amount of) liquidity available in the *future*. The pattern that is emerging is that, rather than arrow colour, the more important semantic qualifier is whether the arrows are solid or dotted. As shown in the centre of Figure 6, solid arrows refer to existing obligations, whereas dotted arrows refer to future commitments. A potential source of confusion is that while set-off in Figure 4 was depicted without a liquidity source, S has appeared in Figure 6a. S could have been just v_3, here, but using S is more general. The simplifying assumption in this first example is that all the amounts (arrow values or weights) are equal – and so do not need to be shown explicitly. The settlement flow is correspondingly trivial and is shown in Figure 7 for the sake of completeness, using green dashed arrows of the same weight.

Figure 6: *Interaction networks for the four ways to settle*

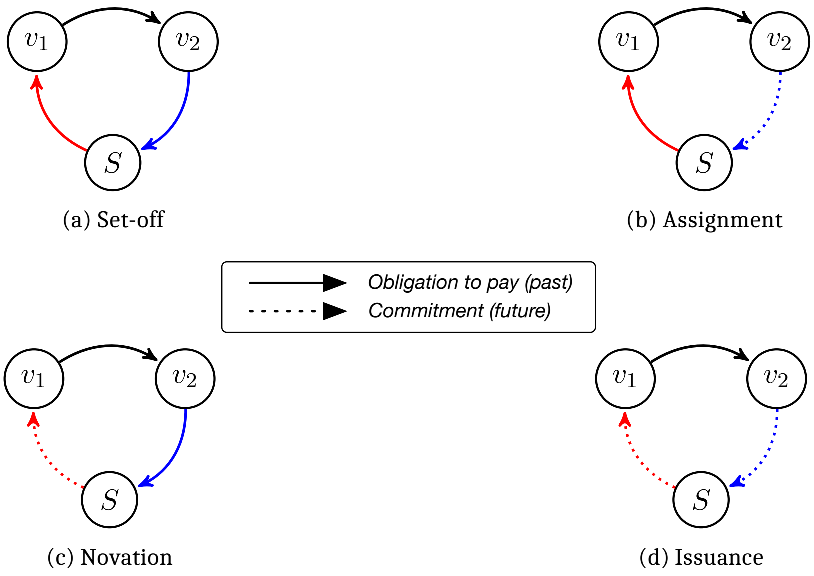
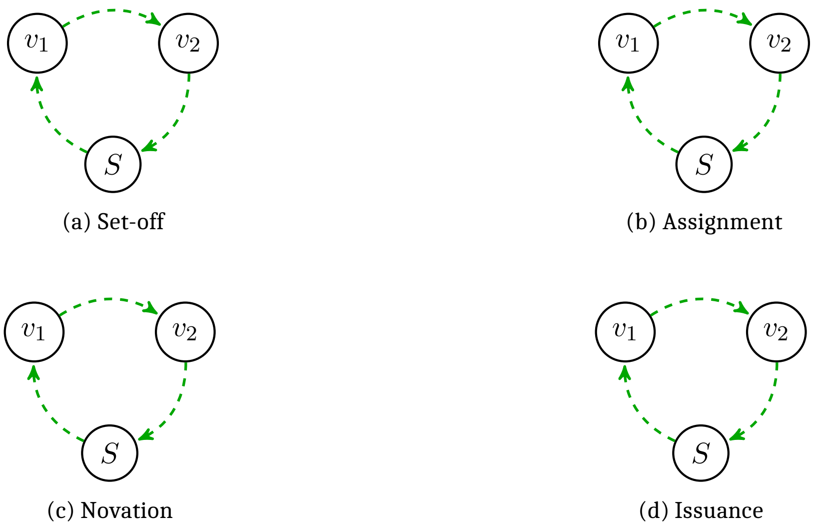


Figure 7: *Settlement flows for the interaction networks of Figure 6*



The last building block of the interaction network flow representation is the network state after the settlement has taken place, i.e. the remaining interaction network. This is shown in Figure 8. Under the assumed simplifying conditions of equal amounts everywhere, after settlement all the obligations necessarily disappear, while the commitments become obligations *but in the opposite direction*:

- An acceptance by v becomes a liability for the bank, i.e. an obligation from the bank to v.
- A credit line towards v becomes a debt, i.e. an obligation from v towards the bank.

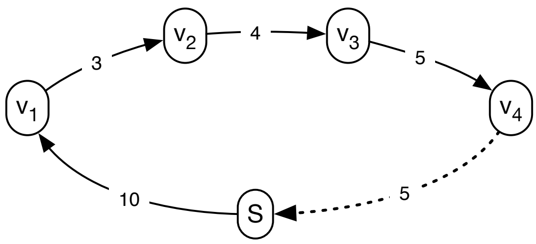
If the arrows have a mixture of different values, then some of the obligation arrows might remain, as in standard set-off. The commitment arrows, on the other hand, will split: the parts that were settled will become obligations and will point in the opposite direction, while any remaining commitment amount will remain a commitment. This is demonstrated in the next example.

Figure 8: *State of the network after settlement: new obligations are formed that replace the commitments but in the opposite direction*

(a) Set-off (b) Assignment (c) Novation (d) Issuance

Figure 9, where we have dropped the colourful arrows, shows an interaction network involving a mixture of set-off (for v_2 and v_3) and assignment (between v_1 and v_4). This example provides a demonstration of how MTCS helps mitigate the late-payment problem. In fact, in the presence of such a chain of three sequential invoices, if there is a general dearth of liquidity (as during a financial crisis) v_4 would need to wait for the payments to propagate downstream along the chain to v_3, and this process could take months depending on the trade credit agreements between the firms. With MTCS, on the other hand, (partial) settlement can occur for all the firms at the same time, as an atomic event, as long as v_1 sends the same amount to v_4 that is set off for v_2 and v_3.

Figure 9: *Example of graphical notation for a case of set-off and assignment*

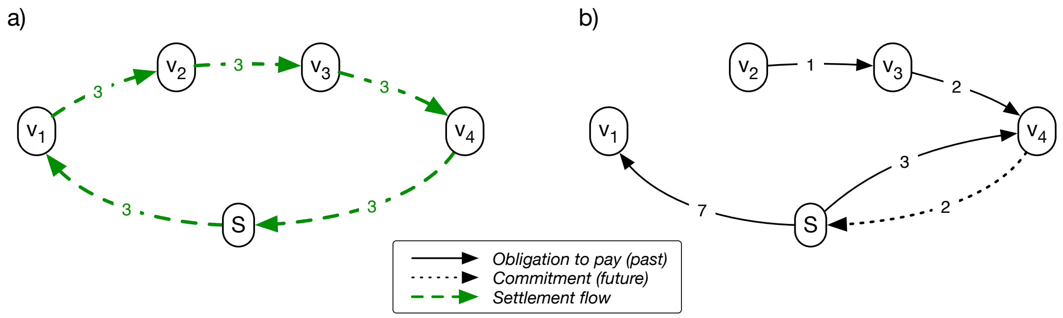


This kind of network that mixes set-off with the use of own liquidity by one or more firms can be confusing when it is first encountered. In fact, it may look like the assignment operation in Figure 9 implies two levels of 'creditor substitution' as described visually in Figure 4b. Since the intuitive perception of this fact may conflict with the understanding of the formal accounting definition of assignment, it is important to clarify this point:

- The first level of perceived creditor substitution – v_1's creditor v_2 substituted with v_4 – is not "real" from a legal perspective. It may look like there has been a change of contract, i.e. novation, but that's not the case. The contract between v_1 and v_2 has not been changed. Rather, all the nodes, including S if it is a bank or other payment system, receive legally-binding set-off notices (UNIDROIT, 2016) that inform v_2 and v_3 about the updated values of their incoming and outgoing invoices due to the set-off; and v_1, v_4, and S that the set-off amount should be moved from v_1's to v_4's account.
- The second level of creditor substitution is the real one: it is the assignment to v_4 of v_1's title to the asset corresponding to the set-off amount. This assignment is performed by the bank.[14]

In this example, the set-off amount is 3 units, as shown in Figure 10a. The result of applying the settlement flow is shown in Figure 10b: the obligation between v_1 and v_2 has disappeared, the other two obligations are smaller by the set-off amount, and v_1's balance at the bank is also smaller by that amount. The commitment from v_4 to the bank was simply an indication that v_4 was willing to accept the 5 units from its debtor v_3. Since the set-off amount was actually sent as an assignment operation from v_1 to v_4, it now sits in v_4's account and, therefore, is shown as a bank's liability – or "obligation" – towards v_4, which remains willing to accept into its bank account the remaining balance of 2 from v_3.

Figure 10: *Settlement flow and remaining interaction network for the Figure 9 example*



We can now provide more precise definitions of the terms we have been using to refer to network flows (see Fleischman and Dini (2026) for more details):

- The *original interaction network* is the input to a minimum-cost maximum-flow solver such as MTCS and represents the initial *network flow* state.
- The *settlement flow* consists of all the minimum-value arrows involving both obligations and commitments that represents the solution of the solver (in Fleischman et al. (2020) and Fleischman and Dini (2021) this was called the cyclic structure). The settlement flow, shown in Figure 10a for this example, is well-balanced by construction, meaning that at each node it satisfies Kirchhoff's first circuit law.
- The *remaining interaction network* is shown in Figure 10b and consists in whatever is left of the original network after the settlement flow solution has been applied, so it is the final network flow state.

The discussion of the ideas presented so far should make Table 1 easier to understand. We thus see that network flow state, and its change as the corresponding settlement flow, is a more general concept and construction than the 'topological change' discussed in the description of Figure 4 because in general it includes both a change in arrow weights and, whenever any commitments are present or obligations are cleared fully, also a change in topology.

4.3.2 The Balance Sheet View

Having introduced a new notation focused on the *edges* of the graph, which represent interactions, it is helpful to provide a link back to the point of view of the *nodes* of the graph, which can be represented by their balance sheets. The graphical representation of financial networks with commitments makes it easy to identify changes in the size of the balance sheets. Figure 11 shows how the introduction of the commitment arrows enables a unique invertible mapping (bijection) between the four ways to settle and the pairs of arrow types entering and leaving a given node. While set-off causes the balance sheet to shrink and issuance to grow, asset substitution and liability substitution do not affect its size. See Fleischman and Dini (2026) for a more detailed discussion of this view.

Figure 12 shows a visualization of what happens to the balance sheets of the counterparties in the four ways to settle, and Table 5 provides the key for the acronym abbreviations.

Figure 11: *Effect of payment/settlement on single firms' balance sheets for the four ways*

Figure 12: *Effect of the four ways to settle on the balance sheets of the counterparties*

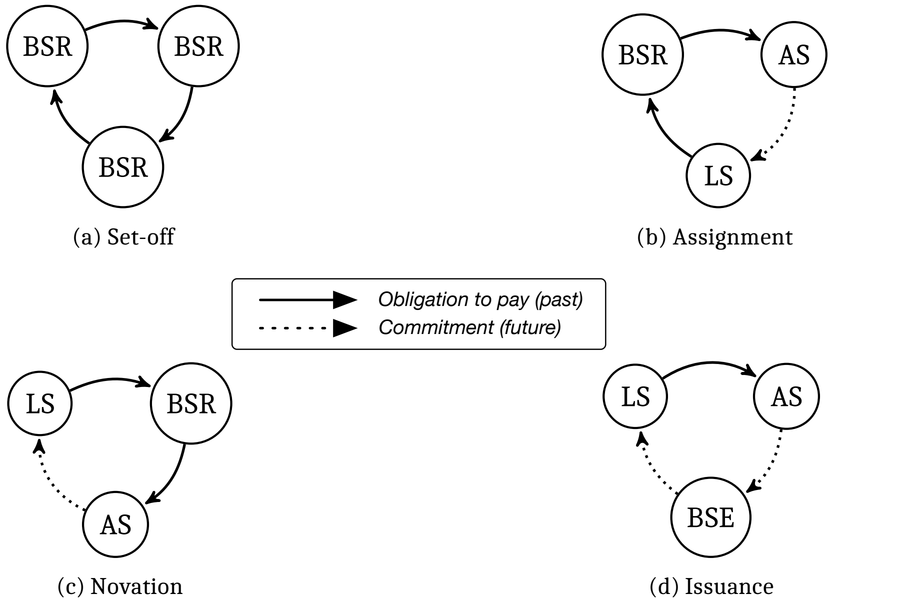


Table 5: *Key for decoding Figure 12*

Abbreviation	Meaning
BSR	Balance Sheet Reduction
BSE	Balance Sheet Expansion
AS	Asset Substitution
LS	Liability Substitution

Having provided all the relevant definitions of the new concepts and the corresponding network flow graphical notation, the next section analyses a relatively simple example in detail.

4.4 Trade Credit Toy Example

The network graphical model just presented in now applied to a toy example involving two communities and four users:

- Community **A** composed of Alice and Alex.
- Community **B** composed of Bob and Bea.

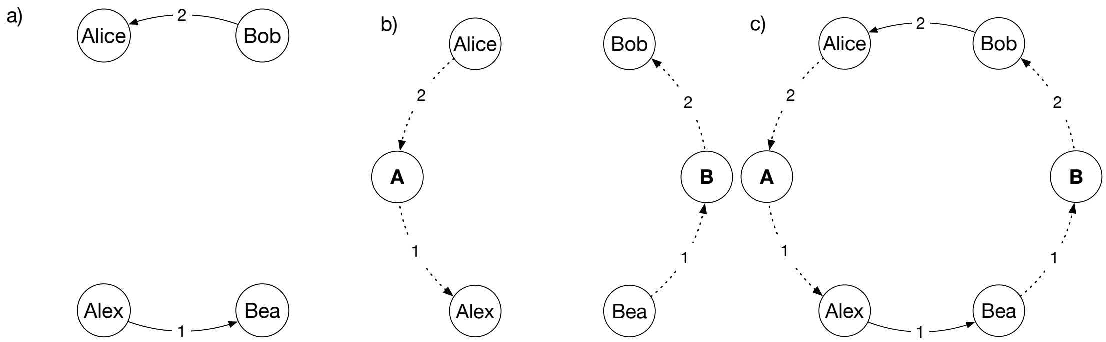
They trade with each other and across communities, after which there are two obligations outstanding (Figure 13a):

- Bob owes 2 to Alice, so Alice has a positive net position of 2 in this example.
- Alex owes 1 to Bea, so Bea has a positive net position of 1.

Each community uses its own mutual credit system, where the name of the mutual credit currency matches the name of the community:

- Alice and Alex use mutual credit system **A**.
- Bob and Bea use mutual credit system **B**.

Figure 13: *Setting up the interaction network of the toy example.*
a) Original obligations; b) commitments; and c) interaction network

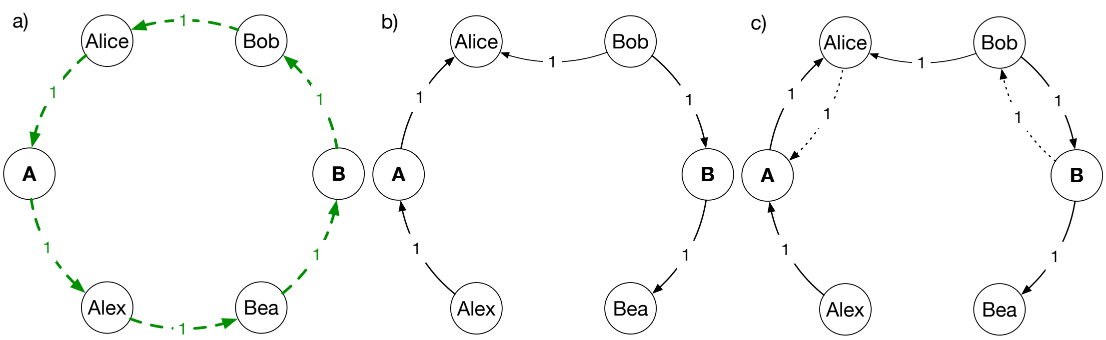


Finally, the two mutual credit systems use the same unit of account. For example, they could be pegged 1-1 to the same fiat currency. The first two examples below demonstrate the positive effects of multilateral settlement as interoperability and financial expression, respectively, while the third example demonstrates the negative effect of liquidity extraction from communities by an external financial actor. More examples are analysed in Fleischman and Dini (2026).

4.4.1 Settlement by Mutual Credit

Everyone is willing to accept (its own) mutual credit. Bob and Alex both have zero balances, at the moment. The reason we know this is that each is declaring an amount from his credit line that he is making available to settle his obligation. We do not know the balances of Alice and Bea, but we know that Alice is willing to accept her positive net position in **A** credits while Bea is willing to accept her positive net position in **B** credits. This is shown in Figure 13b as dotted arrows, i.e. future-oriented commitments. Figure 13c shows the obligations and commitments together as an interaction network.

Figure 14: *Settlement flow, remaining obligations, and remaining interaction network*



The (partial) clearing of such a network is achieved by the settlement flow, which is found by the MTCS minimum-cost maximum-flow solver as a generalization of the search for the closed cycles presented in Fleischman et al. (2020) and Fleischman and Dini (2021). The generalization consists in treating commitments mathematically in the same way as the obligations, i.e. still seeking closed cycles and subtracting the smallest amount from all the interactions regardless of whether they are obligations or commitments. The settlement flow is shown in Figure 14a. This flow looks deceptively simple because it looks like one unit is going around the whole circuit in the same direction and that, therefore, it can be "set off". This, in fact, is a demonstration of the power and generality of the MTCS algorithm because its solution represented by these edges is indeed treated mathematically as no different from set-offs. Their interpretation and practical implementation relative to the four ways to settle, however, requires more careful analysis. In particular, there are two layers of complexity hiding behind Figure 14a:

- Depending on the type of interaction, the meaning of 'settlement' changes:
  - For the obligation between Bob and Alice, *settlement* means *set-off*, and one unit is subtracted from Bob's debt (and from Alice's credit).
  - For Alice's acceptance of **A** units, *settlement* means that a unit *is written into* her account. This is the second half of an assignment.
  - For Alex's credit line in **A** units, *settlement* means that he *issues* one **A** unit.
  - For Alex's debt towards Bea, *settlement* means that it is *set off* and it disappears.
  - For Bea's acceptance, *settlement* means that one **B** unit *is written into* her account. This is the second half of an assignment as well.
  - And for Bob's credit line in **B** units, *settlement* means that he *issues* it.
- The second layer of complexity arises from the fact that Alice only accepts **A** units, and therefore has no use for Bob's **B** unit; and Bea accepts only **B** units and therefore she has no use for Alex's **A** unit. This is resolved by each mutual credit system, in each case, by assignments that keep units within their communities. Thus, the **A** system sends Alex's **A** unit to Alice, while simultanously the **B** system sends Bob's **B** unit to Bea (these are the first half of the two assignments in the previous bullet). These operations are analogous to the assignment between v_1 and v_4 in the Figure 9 example. Another way to think of this form of settlement is that at an abstract level it is the *simultaneous* analogue of the Bill of Exchange convertible instrument from the Renaissance, even though of course the roles of the four counterparties are different.

Figure 14b shows the remaining obligations after the partial settlement:

- For the obligation between Bob and Alice, one unit of debt remains, drawn as a downward-convex arc to distinguish it from the unit that was set off.
- For Alice's acceptance of **A** units, the **A** system now "owes" Alice one unit, i.e. Alice's balance is now +1.
- For Alex's credit line in **A** units, Alex now has a debt of one **A** unit towards the **A** system, i.e. his balance is -1. This is because he had started with a zero balance in his mutual credit account.
- Bea's acceptance within the **B** system is handled like Alice's acceptance was handled in the **A** system.
- And, similarly, now Bob's owes one unit to the **B** system, i.e. his balance is -1.

Finally, Figure 14c shows the remaining interaction network, which is just the "sum" of Figure 14b and the remaining commitments. The "magic" of the settlement flow, therefore, consists in the fact that Alex was able to discharge his obligation to Bea, and Bob discharged part of his indebtedness to Alice, in spite of the fact that both were transacting across different and non-convertible mutual credit systems.

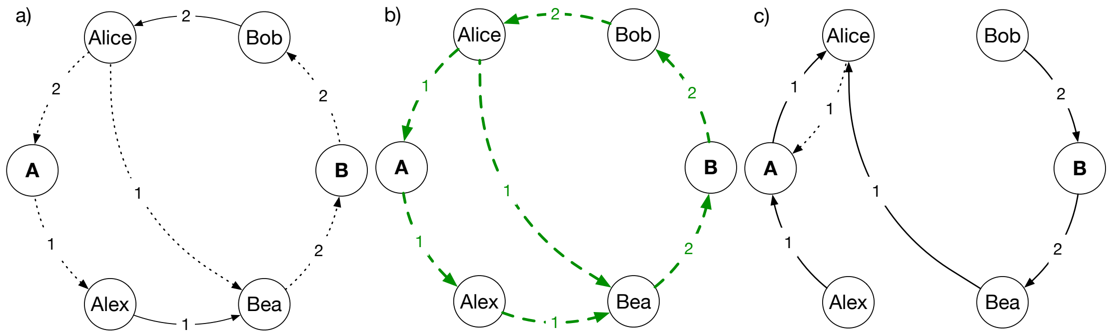
4.4.2 Settlement by Peer-to-Peer Loan

Another option is to create internal liquidity through peer-to-peer (p2p) loans between members of different communities. Let's say Alice and Bea have a long-standing trading relationship. On the strength of that, Bea asks Alice for a credit line. Bea needs a p2p loan from Alice to increase her **B** mutual credit positive balance because she (Bea) is under a deadline to make purchases within the **B** community. This seems counter-intuitive, at first, since Alice is clearly making a loan in **A** units; but it works out through the settlement flow, as follows.

Figure 15a is the same as Figure 13c but with the additional p2p loan from Alice to Bea and a correspondingly larger acceptance from Bea into her **B** account. The settlement flow in this case is almost the same picture, except that the acceptance from Alice to her **A** account is 1 instead of 2 because as we saw above settlement flows must be well-balanced. This solution involves two partially overlapping cycles, so it can be 'set off' – using our older terminology – or 'settled' – using the new one.

The result is shown in Figure 15c, which shows the remaining obligations together with the unused commitments, i.e. the remaining interaction network. The original trade credit obligations Bob to Alice and Alex to Bea are gone. The peer-to-peer loan is now a new obligation from Bea to Alice to repay the loan. The discharge of the obligation from Bea to Alice can be monetary (e.g. $) or non-monetary. It all depends on the agreement between them. How was the settlement achieved? Similarly to Figure 14, Alex sent 1 **A** unit to Alice and Bob sent 2 **B** units to Bea.

Figure 15: *Original interaction network, settlement flow, and remaining network*



This example shows how sharing the intents of a future business transaction (a credit line that becomes a loan) acts as liquidity within the settlement network. It is also 'collaborative' since all four users had to coordinate their payments in a very specific way to make the whole thing work. They do not need to understand all the details, but they do have to subscribe to a CoFi multilateral settlement service that implements this rather sophisticated protocol. If Alice and Bea have an on-going trading relationship, its value to each decreases their motivation to defect.

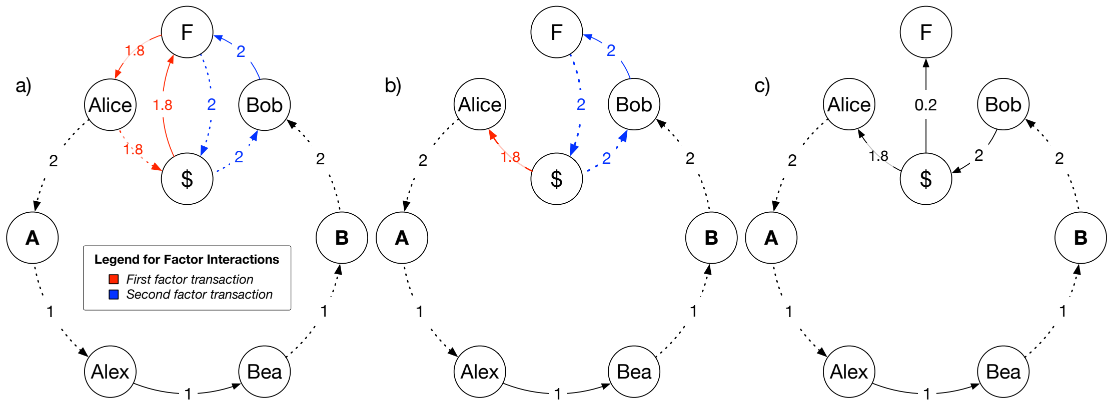
It is fair to say that in general Bea does not know whether or not such a p2p loan from Alice will clear at the next MTCS run, because in general Bea would not be able to tell if Bob has an obligation towards Alice. In general, therefore, such a p2p loan might not participate in the settlement flow at the first try, but with each new MTCS run the probability that it does will increase. The moral is that, where communities have longer-term balanced trades, prudent community members can do finance without (extractive) third parties. Therefore, this is an example of how our accounting and payment system innovations can strengthen the economic sustainability of the weakest members of the network.

4.4.3 Factoring as Example of Value Extraction from Communities

The last example demonstrates that the use of outside money can be negative for communities by looking at a form of intermediation called factoring. Fig. 16a shows a case where Alice is willing to accept less liquidity in order to have it sooner. She relies on a factor (a service provided by many banks) to take on her credit with Bob. The factor becomes Bob's new creditor for that invoice. The interactions as depicted here are a best-case scenario and are simplified for the sake of clarity. Normally, the amount 1.8 that Alice receives is not given all at once. A smaller amount is given by the factor, with the balance contingent on Bob paying on time. If Bob is late, then Alice will be penalized for it at a rate of approximately 30% APR.[15]

Although in principle this factoring case could also have benefited from a settlement flow, the factor's need to break up the payment flow into two separate transactions at two different times makes atomic settlement of the whole network impossible. Fig. 16c shows that the factor's net position at the end of the process is 0.2, which is a lower bound on the value that is extracted from the community. In addition, the inability to use a settlement flow means that the obligation betwen Alex and Bea is still outstanding and may need to be repaid through the banking system.

Figure 16: *Original interaction network, factor transactions, and remaining network*



5. THE BALANCE BETWEEN INSIDE AND OUTSIDE MONEY FOR COMMUNITIES

The previous section has provided two simple and hopefully clear examples of the advantages communities can benefit from if they rely on inside money and one example of the disadvantages they can incur when relying on financial services based on outside money. This section presents the analysis of a large data set of approximately 40,000 Italian companies[16] as a series of simulations[17] that show a similar effect more dramatically. Because the empirical data is protected by privacy guarantees, we used the method described in Berger and Carstens (2018) to generate a 'synthetic' dataset that fits the real-world data. More precisely, the result is a network that has the same network statistics (number of nodes, degree distribution, and obligation amounts) as the real trade credit network, but with a different topology (the edges do not reflect real business relationships).

The object of the simulations is to compare the amount of debt that could be cleared within each community with inside money to the debt across different communities that would normally require outside money. Simultaneous multilateral settlement improves the liquidity-saving for both classes of payments through the different mechanisms explained in the previous sections, for example applying set-off to different currency areas simultaneously. Other kinds of intermediated payment and financial services are of course also possible, but as discussed they imply different levels of liquidity extraction.

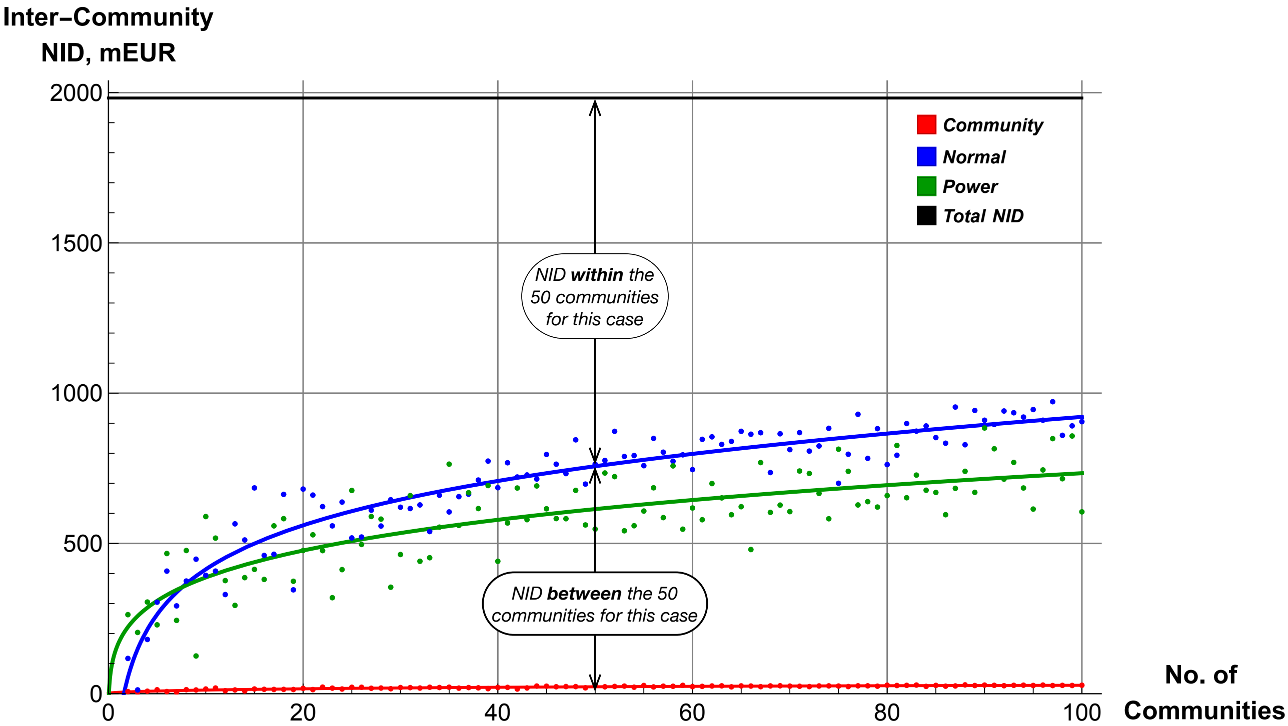
Figure 17 analyses the volume of trade between all the companies over a 1-month period. The total volume of trade or total debt is the sum of all the invoices for the given month. The plot, however, shows the Net Internal Debt (NID), which is the sum or all the positive (or negative) net positions for all the companies. The difference between the total volume and the NID is the amount of liquidity that could be saved by set-off, for example using MTCS.[18] For this dataset, the total NID for the whole network is 1982m Euro. The x-axis counts how many communities N are being used to subdivide the total number of companies for the simulation, where $1 \leq N \leq 100$ and the total number of companies is always 40,000. For each case, the plot compares the total NID, shown as a black horizontal line at the top, to the outside money and inside money NID.

The plot also compares the impact on inter-community and intra-community NID of three different ways in which the communities are constructed:

- **Normal Distribution.** For each number N of communities, $1 \leq N \leq 100$, the algorithm creates a list of probabilities over the N communities that follows a (normalized) Gaussian distribution. The algorithm assigns the 40,000 companies to the communities using these probabilities, i.e. by picking a community for each company (i.e. placing a given company within a community) according to the community's probability. Given the shape of the Gaussian, the communities near $N/2$ have a much higher probability of being chosen than those near 1 or N.

- **Power Distribution.** For each number N of communities, $1 \leq N \leq 100$, the algorithm creates a list of probabilities over the N communities that follows a decreasing, normalized power law ($1/c^\alpha$, $\alpha = 1$ and $1 \leq c \leq N$). The algorithm assigns the 40,000 companies using these probabilities, i.e. by picking a community for each company (i.e. placing a given company within a community) according to the community's probability. Thus, communities near 1 have a much higher probability of being chosen than those near N.
- **Community Distribution.** The Louvain algorithm tries to find groups of nodes that (based on the data) are more densely connected to each other than to the rest of the graph (Traag et al., 2019). These groups are called communities. For example, in a social network, a community might be a group of friends who all know each other. The method works by maximizing modularity, which measures how well the graph is divided into communities. A higher modularity means the algorithm has found good clusters of nodes.

Figure 17: *Variation of inter-community NID with different number N of trading communities for the same set of companies and trades, and with different probability distributions used to construct the communities*



The statistics and probability operations and functions just described may be difficult to understand for some readers. We therefore complement the above descriptions by emphasising that in the normal and power distributions the manner in which the "communities" are built up has *nothing* at all to do with actual trading relationships. Each company is assigned to a given community only based on that community's probability in the chosen distribution. This makes for two artificial and unrealistic networks whose only purpose is to quantify the resulting NID that can be settled with the inside money of each community, within each community. The third or 'community' distribution, which uses the *actual* trading relationships to build up the different communities, results in the largest internal NID by a very large margin.

Figure 17, then, shows a visualization of the effect of the presence of communities on inter-community and intra-community NID for the case where the 40,000 companies are subdivided into 50 communities. The amount of NID that can be settled by inside money is huge even for such an unrealistic network, i.e. approximately 65% of the total. A similar trend is observed for all other values of N, and a similar behaviour results from the Normal and Power distributions. The Community distribution, however, is even more striking. Here we can see that practically *all* the NID can be settled using inside money. This is perhaps to be expected since the red line corresponds to *actual* trading communities. Near the origin all curves go to zero because if there is only 1 community then the inter-community trade is zero by construction. Finally, each curve required a least-squares fit (around a square root-like function) because in such a random simulation the amount of inter-community NID is a discontinuous function of the number of communities. This is true also for the Community case, whose apparent smoothness is just due to the scaling.

This simulation helps highlight the power of neutral settlement networks in two ways. First, since there are many communities some will be very small, such that the utility of their inside money will be similarly limited. But if they participate in the neutral settlement network the ability of their inside money to clear obligations is extended beyond the boundaries of their issuing communities. Second, there is a balance between the amount of inside and outside money that provides best results – although this is still an open research question. Be that as it may, in a world where size matters, the neutral settlement layer presented here is well-suited to supporting a "community of communities" in their adoption of special-purpose community currencies interoperable with each other and with fiat.

6. SUMMARY AND CONCLUSION

The paper started with two research questions:

1. Can multilateral settlement increase the economic impact of local currencies?
2. Do the innovations discussed in the paper strengthen the ability of economic systems to meet sustainability expectations?

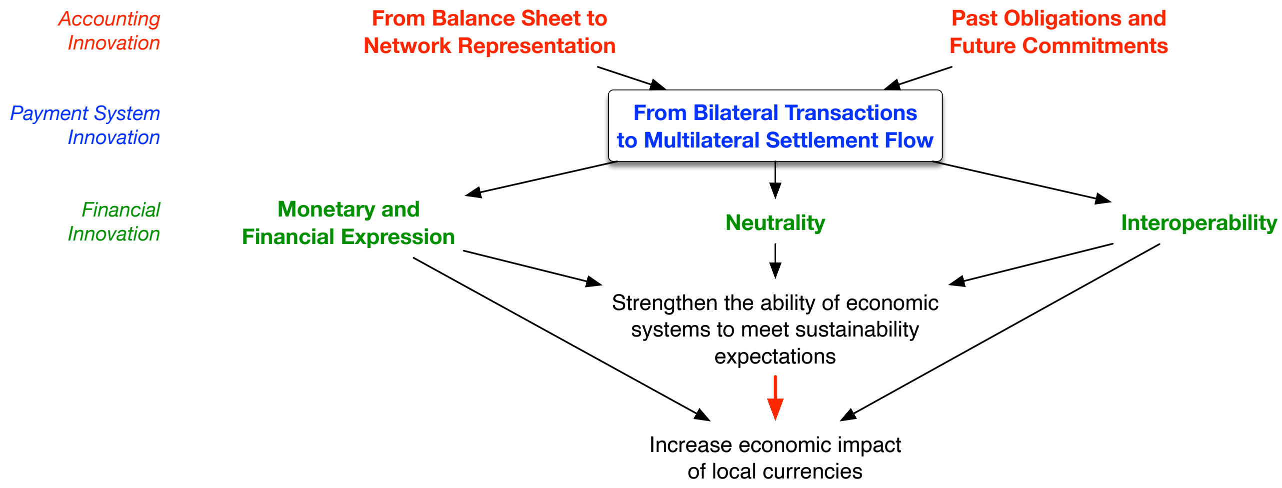
Because a familiar, positivist, empirical, and inductive methodology could not be used to test the impact on real economic systems of a speculative multilateral settlement protocol that has not yet been implemented anywhere, the paper adopted an abstract, deductive methodology to infer the social, political, economic, and financial implications of such an innovation in payment system mechanics.

The abstraction comes from the reliance on a graph theory optimization method based on a network flow algorithm to uncover parts of a given obligation network where liquidity can be saved by set-off, MTCS in this paper. Mathematically, this is made possible by applying deferred multilateral set-off simultaneously to the whole network at fixed intervals of e.g. one month.

From this initial mathematical result, the paper deduced (1) that the same algorithm could be applied simultaneously to future commitments and to past obligations to implement the four ways to settle in a network context, as well as (2) the social, political, economic, and financial significance of these innovations in the machinery of the underlying payment system. Since the mathematical results and their socio-economic significance live in different disciplinary domains, however, an epistemological bridge was needed. This was provided by the language of accounting which, however, *itself* needed to be generalized both by adding future commitments and by extending it from bilateral transactions between balance sheets to a multilateral network flow graphical formalism. Thus, the graphical language was developed to support the presentation of the multilateral settlement protocol as well as the analysis of different network contexts and examples. The political economy and sociological significance could then be inferred through the interpretation of accounting stocks and flows which, therefore, obviated the strict need – although certainly not the desirability – of empirical data for the purpose of reaching preliminary conclusions.

Thus, the paper presented the concept of a multilateral settlement flow as an innovation in the payment system that would not be possible without two underlying innovations in accounting: the introduction of future commitments in addition to existing obligations, and a new graphical notation to represent balance sheet relationships between transacting parties as network edges. The paper argues that these three innovations working together, in turn, lead to significant financial innovations in the form of payment system neutrality, financial expression of liquidity hidden in the topology of the network, and interoperability between different non-convertible mutual credit currencies, fiat, and crypto. As suggested schematically in Figure 18, which extends Figure 3, the paper showed that it is this third layer of innovation which provides a positive *proximate* answer to both research questions, but that such positive effects would not be possible without the underlying accounting and payment-system innovations.

Figure 18: *Schematic showing how the innovations presented answer the research questions posed*



The argument presented starts with a brief historical outline of double-entry book-keeping as a way to explicate the otherwise opaque terminology and mathematical rules of transaction recording used in balance sheet accounting. The motivation was to generalize the baseline reference system that formalizes the accounting of bilateral transactions – set-off, assignment, novation, and issuance – to multilateral settlement in a network context.

To explore the implications of multilateral settlement using network accounting, the paper dedicated significant time and effort to the detailed analysis of a simple network of four nodes and three currencies, representing both inside and outside money, to demonstrate in a transparently verifiable way how a multilateral settlement flow can be applied to different kinds of interaction networks that embody different combinations of the four ways to settle. The toy model demonstrates the claim of financial expression without reliance on third parties, and that the multilateral settlement flow enables different communities utilizing different, non-convertible mutual credit currencies to interoperate as long as there are cycles that straddle the different "currency areas" and a single unit of account is chosen at the time the atomic operation of (partial) settlement is performed.

The neutrality claims are more difficult to prove because of lack of empirical data geared to this specific test. However, the analysis of a much larger network of real companies, suitably anonymized, led to a remarkable result. When using arbitrary probabilistic distributions for defining communities of interacting companies, a very large fraction of the volume of trade across such a real network (50-70%) can be settled using inside money only, as long as there are enough communities using it. But if the communities of interacting companies are derived from the actual data, the fraction reaches above 95%.

The implications for the financial self-determination of communities are significant. As long as each community is able to define its own inside money based on a sufficiently high internal level of trust, the settlement flow is neutral in the sense that it strengthens their ability to interact economically and financially with other communities, the fiat economy, and the cryptosphere on their own merits, counteracting the effects of the market power of the stronger areas of the economy. And the communities' ability to base trade credit relationships on the expression of financial relations such as p2p loans implies a greater ability to take charge of their financial needs, minimizing their reliance on extractive third parties operating in outside money. Therefore, the overall conclusion arising from these insights and the analysis presented by the paper is that the two research questions are not actually independent: sustainability principles act to increase the economic impact of local currencies, as shown by the red arrow in Figure 18.

Notes

1 The issue about small banks is significantly more complex. See Minton et al. (2024); Gopal and Schnabl (2022) for a discussion of the US context. Since this paper is more focused on communities of complementary currencies, further discussion of the very interesting and relevant case of the community banks is left for future research.

2 Defined in this case as transaction volume/monetary mass (Dini and Kioupkiolis, 2019; Simmons et al., 2021).

3 We are indebted to Giuseppe Littera (private communication), one of the co-founders of Sardex, who first formulated this concept to us.

4 WIR = Wirtschaftsring (economic circle in German).

5 One of the anonymous reviewers pointed out that WIR is not properly a mutual credit system. This is true: since 2004 WIR has become 'WIR Bank'. Moreover, in 1938, only four years after its founding, WIR went through a crisis when some of its members defaulted, and had to be recapitalized. At that time, in order to receive permission to continue to function it also had to agree to comply with Swiss banking laws. In 1952 it had to introduce a small level of interest for the larger loans and negative balances. And in 1972 it stopped convertibility with the Swiss Franc to avoid deleterious discounting of the currency. So WIR started out as a "pure" mutual credit system in 1934 but since then it has followed a winding road of adaptation to different economic circumstances that has brought it ever closer to a community bank while retaining the ability of its members to issue credits compatibly with their allocated credit lines. The Sardex system, by contrast, was created explicitly as a copy of the early WIR system. Thus, when we refer to WIR in this paper we mean the early, "pure" version of this very interesting mutual credit/banking phenomenon.

6 Based on Stodder (2009) for the WIR turnover and `https://en.wikipedia.org/wiki/Economy_of_Switzerland` for Swiss economic data.

7 IOU = I Owe You, usually a piece of paper acknowledging a debt.

8 'Atomic events are the basic components out of which composite events can be constructed' `https://www.ibm.com/docs/en/cics-ts/6.x?topic=events-atomic`. In the context of this discussion, 'atomic' refers to the fact that all the obligations are (partially) settled simultaneously, in the same indivisible operation.

9 See also Informal Systems' Cycles White Paper (Buchman et al., 2024) for more discussion of Clavero's model.

10 Although negative numbers were known by Chinese and Indian mathematicians as far back as 2000 years ago, even the famous Arab mathematician Al-Khwarizmi (780-859 AD, from which the word 'algorithm' derives) in his Al-Jabr book (from which 'algebra' comes) shunned them. Until the Renaissance negative numbers were regarded as 'absurd', Fibonacci (1170-1250) apparently being an exception since he used them to indicate a financial debt or loss (`https://en.wikipedia.org/wiki/Negative_number`). In short, from Diophantus to Descartes mathematicians were aware of negative numbers but had a hard time accepting them as legitimate (Stillwell, 2010: 457)

11 The 'net position' of a given company is the difference between its incoming and outgoing cash flows.

12 The Equity column is redundant. Since for multilateral settlement flows $\triangle$Funds and $\triangle$AP/AR are always in the same journal entry, Equity never changes. It is included here for the benefit of non-accountants and to provide continuity with the previous discussion.

13 Although in our previous work (Fleischman et al., 2020; Fleischman and Dini, 2021) we used the terms set-off, tender, and acceptance, it is more general and accurate to talk about an interaction network, which may involve one or more of the four ways to settle discussed.

14 The fact that a bank may be unwilling to implement this protocol and support multilateral trade credit set-off functionality is a fair point even if not directly relevant to the present discussion. It can be addressed by pointing out that a non-profit ethical

or cooperative bank would be highly motivated to support MTCS as it increases the economic viability of its members or that, alternatively, the functionality described is trivial to implement on a blockchain for obligations expressed in e.g. stablecoins.

15 APR = Annual Percentage Rate.

16 The data was obtained from Infocert under NDA (non-disclosure agreement).

17 We should emphasize that these are not simulations of economic *behaviour* but of static economic *relationships* from which behaviour can be inferred based on the ideas in Section 4.

18 Since the effect of settlement by set-off has already been shown and quantified in Fleischman et al. (2020) and Fleischman and Dini (2021), this figure shows something related but slightly different. As we are plotting NID, liquidity-saving by set-off is not shown explicitly. However, if we perform a thought experiment in which all the closed cycles have already been accounted for and set off, applying the simultaneous multilateral settlement flow that the MTCS algorithm implements can still lead to additional liquidity savings if there are any chains of invoices. For example, this is shown in Figure 9, where S can be interpreted as the set of banks for one of the cases shown in Figure 17. In addition, the benefits of interoperability, p2p loans, etc. discussed earlier in the paper would also accrue.

References

Antinori, C. (2004). La contabilità pratica prima di Luca Pacioli: Origine della Partita Doppia. *Revista Española de Historia de la Contabilidad*, 1(1):4–23.

Arrighi, G. (2001). Braudel, Capitalism, and the New Economic Sociology. *Review (Fernand Braudel Center)*, 24(1):107–123.

Bardoscia, M., Barucca, P., Battiston, S., Caccioli, F., Cimini, G., Garlaschelli, D., Saracco, F., Squartini, T., and Caldarelli, G. (2021). The Physics of Financial Networks. *Nature Reviews Physics*, 3(7):490–507.

Berger, A. and Carstens, C. J. (2018). Smaller universes for sampling graphs with fixed degree sequence. ArXiv preprint: https://arxiv.org/pdf/1803.02624.

Blanc, J. (2011). Classifying "CCs": Community, complementary and local currencies' types and generations. *International Journal of Community Currency Research*, 15(4):4–10.

Block, D., Dhaliwal, G. S., Kretz, D., and Wagner, M. (2025). Modernizing the U.S. Dollar for the Future of Digital Networks. Technical report, The Digital Dollar Project, Washington. Available online: https://digitaldollarproject.org/wp-content/uploads/2025/03/Modernizing-the-US-Dollar-for-the-Future-of-Digital-Networks-1.pdf. Accessed on 28 March 2025.

Brunnermeier, M. K., James, H., and Jean-Pierre, L. (2021). The digitalization of money. Technical report, Bank for International Settlements Working Paper 941.

Buchman, E., Dini, P., Ahmed, S., Miller, A., and Fleischman, T. (2024). Cycles Protocol: A Peer-to-Peer Electronic Clearing System. Cycles White Paper, Informal Systems.

Clavero, B. (2022). Money and Hierarchy: Four Ways to Discharge a Payment Obligation. Technical report, Local First CIC, De Monfort University.

Croall, J. (1997). *LETS act locally: The growth of local exchange trading systems*. Calouste Gulbenkian Foundation, London.

Darby, D. (2020). Storing value in a mutual credit world: Chris Cook. Interview: https://www.lowimpact.org/posts/storing-value-in-a-mutual-credit-world-chris-cook.

Darby, D. (2022). Building the energy commons: Marcus Saul of Island Power. Interview: https://www.lowimpact.org/posts/building-the-energy-commons-marcus-saul-of-island-power.

Darby, D., Green, D., and Cook, C. (2023). Housing Commons: Introduction. Blog available at https://www.lowimpact.org/categories/housing-commons.

Dini, P., Iqani, M., and Mansell, R. E. (2011). The (Im)Possibility of Interdisciplinarity: Lessons from Constructing a Theoretical Framework for Digital Ecosystems. *Journal of Culture, Theory and Critique*, 52(1):3–27.

Dini, P. and Kioupkiolis, A. (2019). The alter-politics of complementary currencies: The case of Sardex. *Cogent Social Sciences*, 5(1):1–21. https://www.tandfonline.com/doi/full/10.1080/23311886.2019.1646625.

ECB (2023a). Survey on the access to finance of enterprises in the euro area.

ECB (2023b). Total number of credit institutions legally incorporated in the reporting country.

Eisenberg, L. and Noe, T. H. (2001). Systemic Risk in Financial Systems. *Management Science*, 47(2):236–249.

Eisenstein, C. (2011). *Sacred Economics*. North Atlantic Books.

Ellerman, D. P. (1985). The Mathematics of Double Entry Bookkeeping. *Mathematics Magazine*, 58(4).

Fleischman, T. and Dini, P. (2021). Mathematical Foundations for Balancing the Payment System in the Trade Credit Market. *Journal of Risk and Financial Management*, 14(9):452. Available online: https://www.mdpi.com/1911-8074/14/9/452.

Fleischman, T. and Dini, P. (2026). Neutral Settlement of Obligation and Commitment Networks for Liquidity-Generating and Risk-Reducing Topologies of Traders Using Inside and Outside Money. Forthcoming.

Fleischman, T., Dini, P., and Littera, G. (2020). Liquidity-Saving through Obligation-Clearing and Mutual Credit: An Effective Monetary Innovation for SMEs in Times of Crisis. *Journal of Risk and Financial Management*, 13(12):295. Available online: https://www.mdpi.com/1911-8074/13/12/295.

Geijsbeek, J. (1914). *Ancient double-entry bookkeeping: Lucas Pacioli's treatise (AD 1494, the earliest known writer on bookkeeping) reproduced and translated with reproductions, notes, and abstracts from Manzoni, Pietra, Mainardi, Ympyn, Stevin, and Dafforne*. John Geijsbeek Publishing, Denver, Colorado.

Giaglis, G., Dionysopoulos, L., Charalambous, M., Kostopoulos, N., Vlachos, I., Damvakeraki, T., Noszek, Z., Sovershennyi, A., Chykhradze, K., Papoutsoglou, I., Votis, K., Anania, A., Bonetti, A., Le Hors, A., de Moura, E., Copic, E., Skultetyova, I., Ramaswamy, J., Seigneur, J.-M., Ellul, J., Fantacci, L., Niforos, M., Hammond, M., Machado Ruiz, M., Dini, P., Stornetta, W. S., Loesch, S., Maranhão Moreno, S., and Slapnik, T. (2021). Central Bank Digital Currencies and a Euro for the Future. European Union Blockchain Observatory and Forum Thematic Report https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3891861.

Gómez, G. and Dini, P. (2016). Making sense of a crank case: monetary diversity in Argentina (1999–2003). *Cambridge Journal of Economics*, 40(5):1421–1437.

Gopal, M. and Schnabl, P. (2022). The Rise of Finance Companies and FinTech Lenders in Small Business Lending. *The Review of Financial Studies*, 35(11):4859–4901.

Greco, T. (2009). *The End of Money and the Future of Civilization*. Chelsea Green Publishing.

Kiyotaki, N. and Moore, J. (2000). Inside money and liquidity. Unpublished manuscript, University of Edinburgh. http://www.princeton.edu/~kiyotaki/papers/km4-5.6.2018(1).pdf.

Lagos, R. (2010). Inside and outside money. In *Monetary Economics*, pages 132–136. Palgrave Macmillan.

Littera, G., Sartori, L., Dini, P., and Antoniadis, P. (2017). From an Idea to a Scalable Working Model: Merging

Economic Benefits with Social Value in Sardex. *International Journal of Community Currency Research*, 21:6–21. https://ijccr.files.wordpress.com/2017/02/littera-et-al.pdf.

Minton, B., Taboada, A. G., and Williamson, R. (2024). Is the Decline in the Number of Community Banks Detrimental to Community Economic Development? NBER Working Paper 32521.

Padgett, S. (2013). The Negative Impact of Basel III on Small Business Financing. *Ohio State Entrepreneurial Business Law Journal*, 8(1):183–207.

Piketty, T. (2011). *Capital in the Twenty-First Century*. Harvard University Press, Cambridge, Mass.

Sangster, A. (2025). The Diffusion of Double Entry Bookkeeping before 1800—Establishing a New Research Agenda. *Accounting Historians Journal*, 52(1):95–113.

Sartori, L. and Dini, P. (2016). Sardex, from complementary currency to institution. A micro-macro case study. *Stato e Mercato*, 107:273–304.

Simić, S. and Milanović, V. (1992). Some Remarks on the Problem of Multilateral Compensation. *Publikacije Elektrotehnic kog fakulteta. Serija Matematika*, pages 27–33. Available online: https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.100.8429&rep=rep1&type=pdf (accessed on 23 June 2021).

Simmons, R., Dini, P., Culkin, N., and Littera, G. (2021). Crisis and the Role of Money in the Real and Financial Economies—An Innovative Approach to Monetary Stimulus. *Journal of Risk and Financial Management*, 14(3):129. Available online: https://www.mdpi.com/1911-8074/14/3/129.

Sosnowski, R. (2003). Alcune caratteristiche della lingua dell'economia medievale. In Lamberti, M. and Bizzoni, F., editors, *Italia: Literatura, Pensamiento y Sociedad*, pages 235–246. Facultad de Filosofia y Letras, Universidad Nacional Autonoma de México.

Stevenson, G. (2024). *The Trading Game*. Allen Lane (Penguin), London.

Stillwell, J. (2010). *Mathematics and Its History (3rd Ed.)*. Springer, New York.

Stodder, J. (2009). Complementary credit networks and macroeconomic stability: Switzerland's *Wirtschaftsring*. *Journal of Economic Behavior & Organization*, 72:79–95.

Studer, T. (1998). *WIR and the Swiss National Economy*. WIR Bank, Basel.

Traag, V. A., Waltman, L., and Nees Jan Van Eck, N. J. (2019). From Louvain to Leiden: guaranteeing well-connected communities. *Nature Scientific Reports*, 9(5233):1–12.

UNIDROIT (2016). Principles of International Commercial Contracts, Art. 1.6(2). International Institute for the Unification of Private Law.

Walker, A. (2017). A Hoard of Unpaid Invoices: Dissecting Economies & Private Market Forces to Solve B2B Late Payments. MS Thesis, Rochester Institute of Technology. https://scholarworks.rit.edu/cgi/viewcontent.cgi?article=10760&context=theses.